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NUMBER THEORY AND DIOPHANTINE ANALYSIS.

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148. Proposed by R. D. CARMICHAEL, Anniston, Ala.

Find all the multiply perfect numbers of n different prime factors and of multiplicity $n-1$.

No solution received.

149. Proposed by REV. R. D. CARMICHAEL, Anniston, Ala.

Prove that every prime of the form $4n+1$ may be expressed as the sum of two parts r and s such that r^2+rs+s^2+1 is divisible by the prime.

I. Solution by E. B. ESCOTT, Ann Arbor, Mich.

Let $p=r+s=$a prime of the form $4n+1$. Substitute $s=p-r$ in $r^2+rs+s^2+1\equiv 0$ (mod. p) and we have $r^2+r(p-r)+(p-r)^2+1\equiv 0$ (mod. p) or $r^2+1\equiv 0$ (mod. p), which is always possible when p is a prime of the form $4n+1$, and in many other cases.

Example. If $p=5$, $r^2+1\equiv 0$ (mod. 5) has the solution $r\equiv 2$ (mod. 5). $\therefore r=2$, $s=3$.

II. Solution by E. T. BELL, University of Washington.

When $p\equiv 1$ (mod. 4) : $\left(\dfrac{-1}{p}\right)=1$ (Legendre's symbol).

Hence, for r a root of $x^2\equiv -1$ (mod. p), $rp-(r^2+1)\equiv 0$ (mod. p); whence, putting $s=p-r$, $rs-1\equiv 0$ (mod. p).

Therefore, $p^2-(rs-1)\equiv 0$ (mod. p); or, r^2+rs+s^2+1 is a multiple of p; which establishes the stated theorem.

III. Solution by W. F. KING, Ottawa, Canada.

Since $r+s$ is a prime of form $4n+1$, it may be resolved in one way into the sum of two squares. Suppose it to be so resolved, and to be equal to h^2+k^2.

By the usual process, express h/k as a continued fraction in the form

$$\frac{h}{k}=x+\frac{1}{x_1+}\,\frac{1}{x_2+}\,\frac{1}{x_3+}\cdots$$

Form the successive convergents, and let the convergent next before the last (h/k), be (a/b).

Then $ak-bh=\pm 1$.

And $(ah+bk)^2+(ak-bh)^2=a^2h^2+b^2k^2+a^2k^2+b^2h^2=(a^2+b^2)(h^2+k^2)$.

Therefore, $(ah+bk)^2+(ak-bh)^2$, or $(ah+bk)^2+1$, is divisible by h^2+k^2, or $r+s$.

Now $a<h$, $b<k$. $\therefore ah+bk<h^2+k^2$.

Calling $ah+bk$, r, we have r^2+1, and therefore $r^2+1+s(r+s)$, or r^2+rs+s^2+1, divisible by $r+s$.

It is evident that the number $ah+bk$ which is less than h^2+k^2 and is such that $(ah+bk)^2+1$ is divisible by h^2+k^2, can always be found.

Also solved by G. B. M. Zerr.

PROBLEMS FOR SOLUTION.

ALGEBRA.

303. Proposed by PROF. R. D. CARMICHAEL, Anniston, Ala.

Evaluate the determinant

$$\begin{vmatrix} D_1 & x_1x_2 & x_1x_3 & \dots & x_1x_n \\ x_1x_2 & D_2 & x_2x_3 & \dots & x_2x_n \\ x_1x_3 & x_2x_3 & D_3 & \dots & x_3x_n \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ x_1x_n & x_2x_n & x_3x_n & \dots & D_n \end{vmatrix}$$

304. Proposed by C. N. SCHMALL, New York City.

A policeman on a motor-cycle starts in pursuit of an automobile when the latter has a headway of $\frac{1}{2}$ a mile. A pedestrian who is $\frac{1}{4}$ of a mile ahead of the auto and who is walking at the rate of 5 miles an hour, notices that when the auto overtakes him the policeman is only $\frac{5}{12}$ of a mile behind the auto, and $2\frac{1}{2}$ miles from where the officer started; he overtakes the auto. How long did the chase last?

305. Proposed by S. A. COREY, Hiteman, Iowa.

Prove or disprove, that $\displaystyle\sum_{n=1}^{n=\infty}\frac{1}{(2n-1)^2+4}=\frac{\pi}{4}-\frac{\pi}{8}\left(\frac{\cosh\pi}{\sinh\pi}\right)$.

GEOMETRY.

336. Proposed by F. H. HODGE, The University of Chicago.

A man owning a rectangular field $b=300$ feet by $a=600$ feet, wishes to lay out driveways of equal width having the diagonals of the field as center lines and such that the area of the driveways shall be n/m=one-half, of the area of the field. Determine the width of the driveways.

337. Proposed by T. N. HILDEBRANT, The University of Chicago.

Required the locus of the vertices of the parabolae having a given focus and passing through a given point.

338. Proposed by C. N. SCHMALL, 239 East 7th Street, New York.

Given the base and vertical angle of a triangle, find the locus of the center of its "nine-point" circle. [Ex. 28, p. 65, Casey's *Sequel to Euclid.*]